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FOURTH ORDER MESON EQUATION AND NEUTRON-PROTON SCATTERING

S. P. MISRA

MATHEMATICS DEPARTMENT, RAVENSHAW COLLEGE, CUTTACK-3

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ABSTRACT. We have obtained here the neutron-proton differential scattering cross-section with a fourth order meson equation proposed by Bhabha and Thirring, which was useful in explaining the anomalous magnetic moments of nucleons. We note that for moderate energies the results here disagree as violently with experiments as for conventional meson theory satisfying the Klein-Gordon equation.

INTRODUCTION

It was noted in a previous paper (Misra and Deo, 1956) that the treatment of the anomalous magnetic moments of nucleons with a fourth order meson equation proposed by Bhabha (1950) and Thirring (1950) gives comparatively satisfactory agreement with experimental results While proposing this equation, Bhabha had shown that the second order potential derived from this does not have the r^{-3} singularity Because of this advantage and its previous success, we shall calculate the neutron-proton scattering cross-section in the second order perturbation theory.

The interaction hamiltonian in this case is given as

$$H_i(x) = if\,\bar{\Psi}(x)\gamma_5\tau_k\psi(x)\phi_k(x). \qquad \dots \quad (1)$$

The interaction representation field-operators satisfy the equations

$$(\gamma_\mu\partial_\mu + \kappa_0)\psi(x) = 0,$$
$$\partial_\mu\bar{\Psi}(x)\gamma_\mu - k_0\bar{\Psi}(x) = 0$$

and

$$(\Box - \kappa^2)^2\phi_k(x) = 0$$

The vacuum-expectation values of the P-products of the field operators are

$$<P(\phi_i(x)\phi_j(y)) >_0 = \frac{\kappa^2}{2}\,\delta_{ij}D_F(x-y) \qquad \dots \quad (2)$$

where

$$D_F(x) = -\frac{2i}{(2\pi)^4}\int\frac{\exp\,(ikx)}{(k^2+\kappa^2)^2}\,d^4k \qquad \dots \quad (2a)$$

and

$$<P(\psi(x)\bar{\psi}(y))>_0 = \frac{1}{2} S_F(x-y) \qquad \ldots \quad (3)$$

where

$$S_F(x) = \frac{2i}{(2\pi)^4} \int \frac{i\gamma k + \kappa_0}{(k^2 + \kappa_0^2)} \exp\,(ikx) d^4k \qquad \ldots \quad (3a)$$

As usual, integrals (2a) and (3a) are to be understood in the sense of Feynman.

In the perturbation calculations, the S-matrix is given as

$$S = 1 + \Sigma\, S_n$$

where

$$S_n = (-i)^n/(n!) \int d^4x_1 \ldots d^4x_n P(H_i(x_1)\ \ldots\ H_i(x_n)).$$

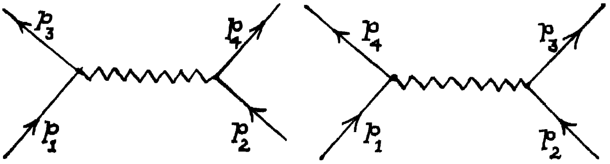
SECOND ORDER S-MATRIX ELEMENT AND NEUTRON-PROTON SCATTERING

In the following, whenever we use the momentum space, p_1 and p_2 denote the four-momenta of the incoming proton and neutron and p_3 and p_4 denote the four-momenta of the outgoing proton and neutron respectively. Also, we use the expansion

$$\psi(x) = \frac{1}{(2\pi)^{3/2}} \int d^3p \sqrt{\frac{\kappa_0}{p_0}} (a_p u(p) e^{ipx} + b_p^* v(p) e^{-ipx}) \qquad \ldots \quad (4)$$

for the Dirac field operator where a_p and b_p^* represent respectively the annihilation and the creation operators of the particles and anti-particles.

The two Feynman diagrams that contribute to the second order matrix element are shown in Figs. 1(a) and 1(b).



(a) 1(b)

Fig. 1

These contribute to the second order matrix element, on using the expansion (4) and the results (2a) and (3), as,

$$<S_2> = -\frac{if^2\kappa^2}{(2\pi)^2} \delta(p_1 + p_2 - p_3 - p_4)\kappa_0^2 (p_{10}p_{20}p_{30}p_{40})^{-\frac{1}{2}}$$

$$|\bar{u}(p_3)\gamma_5\tau_i u(p_1)\bar{u}(p_4)\gamma_5\tau_i u(p_2)((p_1 - p_3)^2 + \kappa^2)^{-2}$$
$$-\bar{u}(p_3)\gamma_5\tau_i u(p_2)\bar{u}(p_4)\gamma_5\tau_i u(p_1)((p_2 - p_3)^2 + \kappa^2)^{-2}]. \quad \dots \quad (5)$$

On carrying out summation over isotopic spin indices for the proton and neutron states, we obtain,

$$<S_2> = \frac{if^2k^2}{(2\pi)^2}\,\delta(p_1 + p_2 - p_3 - p_4)\kappa_0^2(p_{10}p_{20}p_{30}p_{40})^{-\frac{1}{2}}$$
$$|\bar{u}(p_3)\gamma_5 u(p_1)\bar{u}(p_4)\gamma_5 u(p_2)((p_1 - p_3)^2 + \kappa^2)^{-2}$$
$$+ 2\bar{u}(p_3)\gamma_5 u(p_2)\bar{u}(p_4)\gamma_5 u(p_1)((p_2-p_3)^2 + \kappa^2)^{-2}]. \quad \dots \quad (6)$$

Now, we know that if the S-matrix element between the initial state $|i>$ and the final state $|f>$ is given as

$$<S> = \delta(P_i - P_f) <f|M|i> \quad \dots \quad (7)$$

where P_i and P_f are the total initial and final four-momenta respectively, then the total scattering cross-section is given as

$$\sigma = (2\pi)^2 \frac{p_{10}p_{20}}{((p_1p_2)^2-\kappa_0^4)^{\frac{1}{2}}}\, S_i\, S_f\,\{\delta(P_i - P_f)\,|<f|M|i>|^2\}, \quad \dots \quad (8)$$

(Jauch and Rohrlich (1955)), where S_i stands for averaging over the initial states and S_f stands for the summation over all the final states. When we utilise the centre of mass coordinates, the equation (8) gives us the differential scattering cross-section as

$$\frac{d\sigma}{d\Omega} = \pi^2E^2S_i\ \Sigma\ |<f|M|i>|^2, \quad \dots \quad (9)$$

where Σ stands for summation over final spin states.
For our problem, equations (6) and (7) give us

$$<f|M_2|i> = \frac{if^2k^2}{(2\pi)^2}\,\frac{k_0^2}{E^2}\,|\,\bar{u}(p_3)\gamma_5 u(p_1)\bar{u}(p_4)\gamma_5\, u(p_2)\,((p_1-p_3)^2+\kappa^2)^{-2}$$
$$+2\bar{u}(p_3)\gamma_5 u(p_2)\bar{u}(p_4)\gamma_5 u(p_1)((p_2-p_3)^2+\kappa^2)^{-2}]. \quad \dots \quad (10)$$

The spin summation in equation (9) can be performed by using the projection operator $\Lambda_+(p)$ to the positive energy states given as

$$\Lambda_+(p) = \frac{-i\gamma p + \kappa_0}{2E}\,\beta$$

Then we have, for equation (10),

$$S_i \, \Sigma | < f | M | i > |^2$$

$$= \frac{1}{2} \frac{f^4 \kappa^4}{(2\pi)^4} \frac{\kappa_0^4}{E^4} [((p_1 - p_3)^2 + \kappa^2)^{-4} \, Sp(\beta\gamma_5 \wedge_+(p_1)\gamma_5\beta \wedge_+(p_3)] \times$$

$$\times Sp[\beta\gamma_5 \wedge_+(p_2)\gamma_5\beta \wedge_+(p_4)] + 4((p_2 - p_3)^2 + \kappa^2)^{-1} \, Sp[\beta\gamma_5 \wedge_+(p_2)\gamma_5\beta \wedge_+(p_3)] \times$$

$$\times Sp[\beta\gamma_5 \wedge_+(p_1)\gamma_5\beta \wedge_+(p_4)] + 2((p_1 - p_3)^2 + \kappa^2)^{-2} \, ((p_2 - p_3)^2 + \kappa^2)^{-2} \times$$

$$\times \{Sp[\beta\gamma_5 \wedge_+(p_1)\gamma_5\beta \wedge_+ p_4 \beta\gamma_5 \wedge_+(p_2) \, \gamma_5\beta \wedge_+(p_3) + \text{hermitian conjugate expn.}\}]$$

$$= \frac{1}{2} \frac{f^4 \kappa^4}{(2\pi)^4} \frac{\kappa_0^4}{E^8} [((p_1 - p_3)^2 + \kappa^2)^{-4} \, ((p_1 p_3) + \kappa_0^2) \, ((p_2 p_4) + \kappa_0^2)$$

$$+ 4((p_2 - p_3)^2 + \kappa^2)^{-1} \, ((p_2 p_3) + \kappa_0^2) \, ((p_1 p_4) + \kappa_0^2)$$

$$+ ((p_1 - p_3)^2 + \kappa^2)^{-2} \, ((p_2 - p_3)^2 + \kappa^2)^{-2} \{(p_1 p_4)(p_2 p_3) - (p_1 p_2)(p_3 p_4) + (p_1 p_3)(p_2 p_4)$$

$$+ \kappa_0^2((p_1 p_4) - (p_1 p_2) + (p_1 p_3) + (p_2 p_4) + (p_2 p_3) - (p_3 p_4)) + \kappa_0^4\}]. \quad \dots \quad (11)$$

We now remember that for the centre of mass system,

$$p_1 = (\vec{P}, E), \qquad p_2 = (-\vec{P}, E)$$

$$p_3 = (\vec{P'}, E), \qquad p_4 = (-\vec{P'}, E), \quad \dots \quad (12)$$

and substitute

$$\vec{P'} \, \vec{P} = P^2 \cos\theta \quad \dots \quad (13)$$

This gives us, on simplification of the above equation (11) and by equation (9),

$$\frac{d\sigma}{d\Omega}$$

$$= \left(\frac{f^2}{4\pi}\right)^2 \frac{2\kappa^4 \kappa_0^4}{E^6 P^4} \left[\frac{\sin^4 \frac{\theta}{2}}{\left(4\sin^2\frac{\theta}{2} + \lambda\right)^4} + \frac{4\cos^4\frac{\theta}{2}}{\left(4\cos^2\frac{\theta}{2} + \lambda\right)^4} \right.$$

$$\left. - \frac{2\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}}{\left(4\sin^2\frac{\theta}{2} + \lambda\right)^2 \left(4\cos^2\frac{\theta}{2} + \lambda\right)^2} \right] \dots \quad (14)$$

Thus, for moderate energies, the differential scattering cross-section is given as

$$\frac{d\sigma}{d\Omega} = \left(\frac{f^2}{4\pi}\right)^2 \frac{2}{k_0^2} \, g(\theta)$$

where

$$g(\theta) = \lambda^2 \left[\frac{\sin^4 \frac{\theta}{2}}{\left(4 \sin^2 \frac{\theta}{2} + \lambda\right)^4} + \frac{4 \cos^4 \frac{\theta}{2}}{\left(4 \cos^2 \frac{\theta}{2} + \lambda\right)^4} - \frac{2 \sin^2 \frac{\theta}{2} \cos^2 \frac{\theta}{2}}{\left(4 \sin^2 \frac{\theta}{2} + \lambda\right)^2 \left(4 \cos^2 \frac{\theta}{2} + \lambda\right)^2} \right],$$

with $\lambda = (\kappa^2/P^2)$. The laboratory energy is given here approximately as $(42/\lambda)$ Mev.

Except for a multiplicative constant, the function $g(\theta)$ above gives us the cross-section, and we have plotted this against θ for $\lambda = 1$ and $\lambda = (1/2)$, i.e., for laboratory energies 42 MeV and 84 MeV respectively (Fig. 2). It is noted that the nature of the curve obtained does not agree with the well-known experimental form, which should be rather symmetrical about 90° with a minimum at slightly less than that value. On the other hand, up to angle 140° the angular distribution is rather suggestive of the behaviour at higher energies. This leads to the suspicion that a smaller value of $(1/\lambda)$ may give the experimental type of curve for lower energies But in such cases the peak of the curve comes too near 90° for these to have any significance.

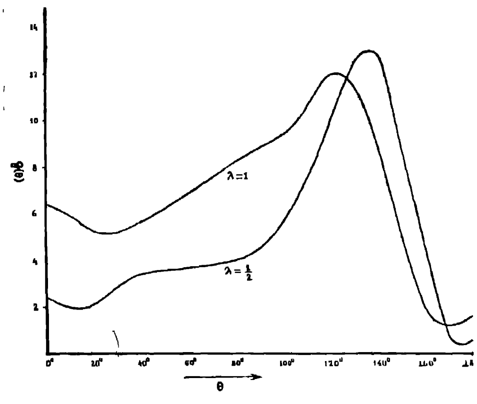


Fig. 2. The function of $g(\theta)$ is plotted against the centre of mass angle θ at laboratory energies 42 Mev ($\lambda = 1$) and 84 Mev ($\lambda = 0{\cdot}5$).

A rough comparison of the scattering data with experiments gives, (on using 1 Mev^{-1} = 1.973 10^{-11} cms), that,

$$\frac{f^2}{4\pi} \simeq 16$$

Thus we note that this meson theory disagrees with experimental results for neutron-proton scattering as violently as the conventional one. Hence the fairly good agreement (Misra and Deo,) (1955)* that was obtained for the treatment of the anomalous magnetic moments of nucleons may be regarded as accidental.

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*There has been unfortunately a mistake in this paper in taking the meson field Lagrangian. This, as taken in the paper, should be multiplied by 2 so that it gives rise to the propagator quoted. This correction also gives better agreement with experiments with $|\Delta\mu_N/\Delta\mu_P| = 1.6$ instead of 2.5.